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Stopping power of an electron gas for antiprotons at intermediate velocities

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The stopping power of antiprotons moving at energies up to 400 keV through a uniform zero-temperature electron gas is calculated within the framework of the kinetic theory. The momentum-transfer cross section required is determined with the aid of a parametric velocity-dependent scattering potential. A nonperturbative constraint, provided by the nuclear-cusp condition, is used to fix the parameter value. A comparison with the result of a quadratic-response-function approach is made. The calculated stopping powers are in good agreement with recent experimental predictions.

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I. INTRODUCTION

The stopping power of condensed matter for massive negatively charged particles ($\pi^-, K^-, \mu^-, \bar{p}$), of interest since the early work of Fermi and Teller [1], has attracted renewed attention due to the availability of beams of antiprotons. The study of the dependence on the sign of the projectile potential is of significance from the viewpoint of a perturbational treatment of the slowing-down process since even and odd orders of the Born series have opposite parity under charge conjugation.

Recent experiments, using the Low Energy Antiproton Ring (LEAR) facility at CERN, have demonstrated marked differences in stopping power for protons p and antiprotons $\bar{p}$ in silicon [2] and gold [3] targets at bombarding energies E down to 200 keV. It should be mentioned that, unlike for antiprotons, the proton data incorporate a charge-state effect [3].

The interaction of the projectile with the constituents of the dense medium has a dynamic many-body character. A simplified electron-gas model provides a standard way for the description. In this model, the energy loss suffered by the projectile can be viewed in terms of free-electron-like excitations induced by the dynamically screened projectile potential.

One can treat the response of the electron gas self-consistently within a mean-field-theory approximation. In linear-response theory [4], we calculate the stopping power to lowest order in the perturbing potential. This theory leads to a stopping power proportional to the square of the projectile charge Z, therefore the experimentally verified differences under charge conjugation can be explained as higher-order, i.e., nonlinear contributions. Unfortunately, the determination of higher-order response functions is difficult [5] and, at present, the calculations are restricted to the quadratic-response level [6,7]. Furthermore, additional approximations regarding the electron gas are made. Hu and Zaremba [6] performed a calculation within the random-phase approximation (RPA), which is rigorous in the high-density limit. Esbensen and Sigmund [7] used the static-electron-gas picture (bosonic description) and obtained results in the so-called quadratic plasmon-pole approximation. These expansion techniques rapidly become unmanageable, and the degree of improvement achieved is difficult to establish [6] due to the lack of a clear convergence criterion.

An alternative approach to nonlinearities is provided by the potential-scattering description within the kinetic theory [8–11] of the stopping-power problem. In the kinetic theory, one postulates that the main mechanism for energy dissipation is basically scattering, and the interaction between the external and system particles is effective only close to the heavy intruder [12]. One advantage of the scattering-theory approach is that it is inherently correct to all orders in the projectile charge Z. The main question that remains in the above framework is the determination of the screened scattering potential. For low-velocity v projectiles, this potential is obviously spherically symmetric. Recent results [13–17] obtained within self-consistent density-functional theory (DFT) demonstrate the importance of strong nonlinearities in the stopping power at the static, i.e., impurity limit.

In this paper, we investigate the capability of the kinetic-theory approach for the case of antiprotons $\bar{p}$ in the bombarding energy range $0 \leq E \leq 400$ keV. We model the interaction via a velocity-dependent spherically symmetric scattering potential. The organization of the paper is as follows. In Sec. II, we provide a brief description of the basic expression of the kinetic theory for the stopping power. The construction of the effective scattering potential is formulated in Sec. III. The numerical results are presented in Sec. IV, and the paper ends with detailed comment in Sec. V. We use atomic units throughout this work.

II. THE STOPPING POWER

The independent-particle model of the system leads to the applicability of the binary-encounter approximation (BEA) of the kinetic theory [8–11] to characterize the energy-loss process. The collisions of electrons with the projectile are considered essentially instantaneous, so their effect can be taken into account using asymptotic properties, e.g., the scattering matrix only [11]. The energy loss per unit path length (dE/dx) for a massive projectile moving with velocity v in a zero-temperature electron gas of density n_0 can be written as [8,10]

$$\frac{dE}{dx}=\frac{1}{(2\pi v)^2}\int_0^{v_F} dv_e v_e \int_{|v-v_e|}^{v+v_e} dv_r v_r^2 (v_r^2+v^2-v_e^2) \times \sigma_{\mathrm{tr}}(v_r) , \tag{2.1}$$

where v_e is the electron velocity, v_F is the Fermi velocity determined from the density as $v_F=(3\pi^2 n_0)^{1/3}$, and $\sigma_{\mathrm{tr}}(v_r)$ is the momentum-transfer (first-order transport) cross section evaluated at relative velocities v_r. In a partial-wave-expansion description of the elastic scattering, the transport cross section is given by [18]

$$\sigma_{\mathrm{tr}}(v_r)=\frac{4\pi}{v_r^2}\sum_{l=0}^{\infty}(l+1)\sin^2[\delta_l(v_r)-\delta_{l+1}(v_r)] , \tag{2.2}$$

in which δ_l's are the phase shifts generated by a spherically symmetric scattering potential. The expressions for the stopping power are particularly simple in the limiting cases [8,11]

$$\frac{dE}{dx}=\begin{cases} n_0 v v_F \sigma_{\mathrm{tr}}(v_F) & \text{for } v \ll v_F \quad (2.3) \\ n_0 v^2 \sigma_{\mathrm{tr}}(v) & \text{for } v \gg v_F . \quad (2.4) \end{cases}$$

In the kinetic theory, an appreciable part of the physics can be found in the dressing of a bare intruder [15].

III. THE SCATTERING POTENTIAL

It is well known that there is a broad overlapping region between the linear-response-function (dielectric) and the kinetic treatments of the stopping-power problem in the Bethe limit, leading to a unified description of the energy loss in terms of a velocity-dependent Coulomb logarithm [8,9]. The equivalence of dielectric and kinetic treatments, at the quadratic-response and second-order Born level, was demonstrated in Ref. [6] for the low-velocity, i.e., static limit.

Our philosophy will be to treat the velocity-dependent effective interaction between an antiproton $Z=-1$ and electrons in an approximation that takes into account the nonlinearity and, retaining a spherical symmetry of the scattering potential, reproduces the asymptotic limits of the stopping power. Let us consider the real part of the RPA dielectric function [19]

$$\mathrm{Re}\epsilon(q,\omega)=1+v(q)\left[\frac{v_F}{\pi}\right]^2\frac{1}{2Q}[\omega_1 f(\omega_1)-\omega_2 f(\omega_2)] , \tag{3.1}$$

where $v(q)=4\pi/q^2$ is the Fourier transform of a bare Coulomb potential, $\omega_1=(\omega+Q^2/2)/Qv_F$, $\omega_2=(\omega-Q^2/2)/Qv_F$, $\omega=\mathbf{q}\cdot\mathbf{v}$, and $f(x)$ is given by

$$f(x)=1+\frac{1-x^2}{2x}\ln\left|\frac{1+x}{1-x}\right| . \tag{3.2}$$

In the propagator part, Q is the transferred impulse, and in the first-order Born approximation for elastic scattering, $Q=q$. Here we use the first two-sided Padé approximant for the $f(x)$ function $[f(x)=2/(1+3x^2)]$ and, in order to characterize nonlinearities, we introduce a one-parameter transformation $Q=\lambda^{1/2}q$. (The actual value of the parameter λ will be determined below.) From the small-ω expansion of the Padé approximant, with the aid of an angle-averaged value of ω^2 $(\omega^2=q^2v^2/3)$, we arrive at the following expression for the dielectric function

$$\mathrm{Re}\epsilon(q,v)=1+\frac{4\pi}{q^2}\frac{n_0}{c+(\lambda q^2/4)} , \tag{3.3}$$

in which $c=(v_F^2+3v^2/\lambda)/3$. We note that for $v=0$, Eq. (3.3) gives the well-known [20,21] result of the linearized Thomas-Fermi-Weizsäcker (TFW-λ) approach for impurity problems. Although the form in Eq. (3.3) refers to the small-ω (therefore small-v) expansion, we use it for arbitrary velocity of the projectile to construct a spherically symmetric scattering potential and induced screening density. (Justifications of this assumption are given below.)

The bare Coulomb potential $-Z/r$ of the antiproton is screened according to

$$V(q)=-Z\frac{4\pi}{q^2\,\mathrm{Re}\epsilon(q,v)} . \tag{3.4}$$

The induced hole density $[\Delta n(q)]$ is then calculated from the Poisson equation to be

$$\Delta n(q)=Z\frac{\mathrm{Re}\epsilon(q,v)-1}{\mathrm{Re}\epsilon(q,v)} . \tag{3.5}$$

The analytic expressions for the real-space equivalents are given by

$$V(r)=-\frac{Z}{r}e^{-\alpha r}\left[\frac{(\alpha+\beta)^2}{4\alpha\beta}e^{+\beta r}-\frac{(\alpha-\beta)^2}{4\alpha\beta}e^{-\beta r}\right] , \tag{3.6}$$

$$\Delta n(r)=Z\frac{n_0}{\lambda\alpha\beta}\frac{e^{-\alpha r}}{r}(e^{+\beta r}-e^{-\beta r}) . \tag{3.7}$$

In the above expressions, the parameters α and β are defined as follows:

$$\alpha=(c/\lambda+\omega_p/\sqrt{\lambda})^{1/2} , \tag{3.8}$$

$$\beta=(c/\lambda-\omega_p/\sqrt{\lambda})^{1/2} , \tag{3.9}$$

where $\omega_p=(4\pi n_0)^{1/2}$ is the classical plasma frequency. The free parameter λ is determined via the constraint provided by the nuclear-cusp condition [22,23] for the total electron density $n(r)=n_0+\Delta n(r)$ at the site of the projectile

$$\left.\frac{n'(r)}{n(r)}\right|_{r=0}=-2Z . \tag{3.10}$$

In the present framework, $\Delta n'(r=0)=-2Zn_0/\lambda$ and thus, for $\Delta n/n_0 \rightarrow 0$, the parameter λ tends to unity. With Eq. (3.10), we can satisfy the obvious requirement $|\Delta n(0)| \leq n_0$ for $Z=-1$.

Let us consider the limiting cases. The velocity-independent version ($c=v_F^2/3$) of the above theory was used in our previous calculation for low-velocity ($v \ll v_F$) antiprotons ($Z=-1$). The results obtained [24,25] for the stopping power and induced hole density are in good agreement with those of a self-consistent density-functional theory [16]. Thus, in a general velocity-dependent problem, one of the desired limits is properly described. Now we discuss the opposite $v \gg v_F$ limit. It is easy to show that the induced hole density [see Eq. (3.7)] at the projectile site has the required Coulombic behavior $\Delta n(0) \sim Zn_0/v$. This limiting behavior is independent of the statistics of the system [26,27]. The Hartree part of the scattering potential close to the origin ($r \rightarrow 0$) is characterized by $\gamma=(\beta^2-\alpha^2)/2\alpha$ [see Eq. (3.6)]; thus, the velocity dependence of the Coulomb logarithm [$L_C=\ln(2v/\gamma)$] is reproduced $L_C \rightarrow \ln(2v^2/\omega_p)$ in the perturbative Bethe limit [4,8] of the stopping theory.

The practical treatment is as follows. For fixed values of v and n_0, we determine the cusp parameter λ from Eq. (3.10) with the aid of Eq. (3.7). Using the analytic parametric expression for the effective potential, we calculate from the scattering Schrödinger equation the phase shifts for fixed values of v_r and the transport cross section from Eq. (2.2). Applying Eq. (2.1), we do numerical integrations.

IV. RESULTS

We consider the stopping power of antiprotons in an electron gas within the framework of Sec. II by choosing the velocity-dependent effective potential of Sec. III. The bombarding energy and velocity ranges of the projectile are $0 \leq E \leq 400$ keV and $0 \leq v \leq 4$, respectively. Comparisons with results of recent experiments [2,3] and the theoretical prediction of a quadratic-response-function theory [7] are also made. In order to avoid uncertainties caused by the finite thickness of the stopping material, we choose experimental points [2,3] that refer to the thinnest targets.

In Fig. 1, we compare theoretical results obtained in our kinetic-theory approach for the stopping power as a function of the projectile velocity v with data of Ref. [2] measured for a Si target. The theoretical curves were calculated for velocity-dependent (full line) and velocity-independent (dashed line) potentials; the latter potential is fixed by the static "impurity model." The Fermi velocity of the system is $v_F=1$, which corresponds to four valence electrons per Si atom. A nice agreement between the experiment [2] and our theory is established. Notice that the experimental points were published without error bars.

Figure 2 shows our prediction for the stopping power in a Au target. The full line refers to the velocity-dependent scattering potential, and the data are taken from Ref. [3]. The filled circles and square are measured values (with error bars) for 1-μm and 2-μm gold foils, respectively. The Fermi velocity is $v_F=1.3$, determined from the measured plasma frequency. A similar conclusion to that obtained at Fig. 1 can be drawn.

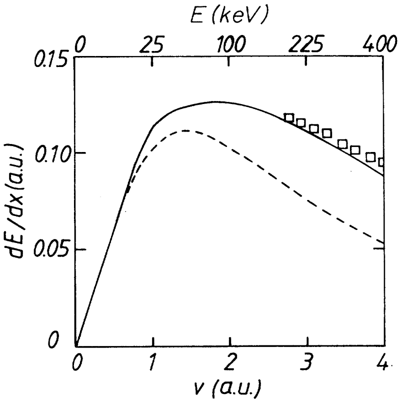

FIG. 1. Stopping power of an electron gas with Fermi velocity $v_F=1$ for antiprotons as a function of the projectile velocity. ——, present work for velocity-dependent scattering potential; — — —, present work for static scattering potential. The open squares represent the experimental data of Ref. [2] measured for a Si target.

It is worth mentioning that the special phase-shift sum [denoted by $C(v_r)$, thus $\sigma_{\rm tr}(v_r)=4\pi C(v_r)/v_r^2$] in Eq. (2.2) is not a strong function of the relative velocity v_r for the case of antiprotons. This is partly due to the repulsive nature of the scattering potential and the extension of the induced screening hole. In fact, we can state that a simple ansatz $C(v_r) \equiv C(v_F)$ in Eq. (2.1) yields fairly good values for the stopping powers in the velocity range $0 \leq v \leq 2v_F$. By this ansatz, the integrations in the general stopping-power expression are straightforward, and we obtain

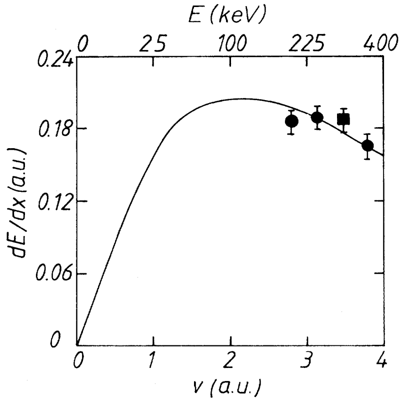

FIG. 2. Stopping power of an electron gas with Fermi velocity $v_F=1.3$ for antiprotons as a function of the projectile velocity. The curve is our result obtained for a velocity-dependent scattering potential. The filled circles and squares are experimental data of Ref. [3] measured for Au targets.

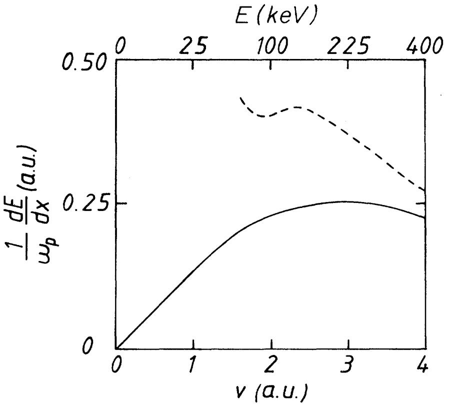

FIG. 3. Reduced stopping power of an electron gas with Fermi velocity $v_F=2$ for antiprotons as a function of the projectile velocity. ——, present work for velocity-dependent scattering potential; — — —, result of a quadratic-response-function theory of Ref. [7].

$$\frac{dE}{dx}=\begin{cases} n_0 v v_F\left[1-\frac{1}{5}\left[\frac{v}{v_F}\right]^2\right]\frac{4\pi}{v_F^2}C(v_F) \quad \text{for } v\le v_F\ , & (4.1)\\ n_0 v^2\left[1-\frac{1}{5}\left[\frac{v_F}{v}\right]^2\right]\frac{4\pi}{v^2}C(v_F) \quad \text{for } v\ge v_F\ . & (4.2)\end{cases}$$

We stress that the upper limit for the "validity" of the latter expression [Eq. (4.2)] is about $v\approx 2v_F$. Equations (4.1) and (4.2) might be useful in antiproton stopping calculations to obtain exploratory results at different target densities, once the static values of $C(v_F)$ are determined [16,24] for "impurity potentials." Equation (4.2) explains the experimentally predicted [3] plateau behavior of the antiproton stopping power at intermediate velocities.

We next compare two theoretical predictions in Fig. 3 for the reduced stopping power $\omega_p^{-1}(dE/dx)$ as a function of the velocity of the projectile. The Fermi velocity of the electron gas is $v_F=2$. The full line represents the result obtained in the framework of the kinetic theory for our velocity-dependent scattering potential. The dashed line is the result of Esbensen and Sigmund [7], who performed the calculation in the quadratic plasmon-pole approximation using the static electron-gas picture. Our curve (which is essentially rigorous for low velocities) tends very slowly to their result. The deviation becomes small only in the high-velocity Bethe range and the two curves merge asymptotically. This observation clearly shows the restricted applicability of a perturbative approach, in agreement with the forecast of Ref. [7].

V. COMMENTS

A scattering approach to the stopping power has been presented. The intrinsic difficulty of the problem resides in the dynamic many-body character of the interaction of the projectile with the electrons and nuclei of the stopping material. In our simplified description of the target as an electron-gas model, the electrons respond collectively to the perturbing potential. This model, although it neglects the detailed atomic structure of a real solid target, provides a reasonable description of electronic stopping at intermediate velocities where the energy dissipation is dominated by loosely bound valence electrons (see Refs. [29] and [30] for proton stopping calculations).

The stopping of a point charge in an electron gas can be described at different levels of sophistication. The standard linear-response theory [4] yields an asymptotic equipartition of individual and collective losses in the average stopping at high velocities. It leads to a stopping power proportional to the square of the projectile charge at arbitrary velocities. However, it was shown [8,9] that the kinetic approach, which is based on the assumption of scattering of individual electrons on the Hartree potential set up by the penetrating ion, correctly describes the asymptotic value of the average energy loss. This approach is inherently correct to all orders in the projectile charge. In the present paper, we have applied the kinetic theory to calculate antiproton stopping power in the intermediate-energy range. The screened scattering potential of the intruder in the electron gas is characterized by a spherically symmetric velocity-dependent form obtained from a self-consistent pseudo-linear-response treatment. Comparisons with measured antiproton stopping powers are presented in Figs. 1 and 2. These results show that our description provides a reasonable characterization of the theoretically and experimentally challenging problem of average energy loss at intermediate velocities.

In our description, the charge-sign effect in the stopping power arises from the nonlinear nature of screening and the nonperturbative treatment of scattering. For a purely atomic model of the target, the effect arises from the binding force that influences the motion of the target electrons during the collision [31]. For velocities higher than that investigated here, the characterization of stopping in condensed matter must unify both of the above mentioned aspects. The small but systematically growing deviations in Figs. 1 and 2 between the theoretical curves and the experimental data above $v\cong 3$ indicate the "opening" of core-electron channels. Certainly in the very high-velocity limit, all electrons are available for stopping, and the asymptotic Bethe expression for the Coulomb logarithm will include the average ionization potential of a target atom [30].

It should be mentioned that for the electron-gas model, the nonperturbative treatment of scattering is of primary interest [15]. Even for a fixed (e.g., linear) screening function in the scattering potentials, the phase-shift analysis gives higher stopping power for protons than for antiprotons [32,33]. On the other hand, the charge-changing channel (therefore the true charge state and its occupancy) may play an important role [17,28,34] for positive projectiles at intermediate velocities. The incorporation of this channel into the applied kinetic theory needs further investigation.

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