

# Coherent states and the interaction of swift ions with condensed matter

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**Abstract** A swift ion moving through an electron gas is described in terms of the coherent states of the interacting system. This formulation is shown to lead simply to well known results for the wake potential generated in the medium by the passage of the ion and for the stopping power of a medium due to plasmon excitation.

**Resumen** Elektroi-gas batean barrena doan ioi laster bat elkarrekintza-sistemako egoera koherenteen arabera azalpidetzen da. Bertan adierazten denez, nolana ere, azalpide honek emaitza ongi ezagunetara eramaten du, bai ioi hau igarotzean bere ingurunean sortzen den wake-potenzialari dagozkionetan eta bai plasmokitzikadurak eraginik inguruneak duen gerarazmenari buruzkoetan.

## 1. Introduction

Phenomena involving the interaction of swift charged particles with matter have been of great importance in the development of modern physics. Bohr's pioneering paper (Bohr 1913) in this area was the first he wrote after his explanation of the spectrum of the hydrogen atom. He remained interested in this field of research all his life. His famous 1948 review paper (Bohr 1948) gives a characteristically thorough exposition of penetration phenomena, treating the complementarity of classical and quantal effects in an especially clear manner. Surveys of experimental and theoretical aspects of charged-particle energy-loss phenomena can be found in a recent review article (Ahlen 1980) or in textbooks (see e.g. Jackson 1975).

More recently, interest has shifted from studies of just the energy loss of the incident particle to the response of the medium during the particle's traversal. The detailed structure of such charged-particle tracks is of great interest in areas such as radiation biology (Mozumder 1969) and in the physics of swift ions in condensed matter (Echenique *et al* 1979). As a fast charged particle moves through a medium, a wake of polarisation may accompany the particle. Theoretical studies of the wake potential, the potential associated with the

dynamic polarisation, have stimulated much experimental work in this area (Remillieux 1980, Gemmell 1980). A recent theoretical description of the wake potential by Echenique *et al* (1979) employs a quantum dielectric function to represent the dynamic response of the medium and involves the use of statistical averages of quantities that are subject to quantal fluctuations.

The coherent-state formalism was developed by Glauber (1963a, b) to describe systems with a large number of photons where one needs to deal with photon statistics and correlations. It provides a fully quantum mechanical treatment of problems in the field of optics which are usually treated classically. The theory can also be applied to other boson fields such as the plasmon field (plasmons are the quanta associated with the polarisation in a medium). The purpose of this article is to illustrate how the wake potential of a fast ion in a model free-electron gas can be derived simply in terms of Glauber coherent states. We recover the standard result for the wake potential and are able to treat quantal effects *a priori* in our development. The plasmon field is excited to a set of coherent states due to the influence of the swift ion. The wake potential is the mean value of the scalar electric potential operator

with respect to the exact state function of the perturbed system. In addition, we calculate the stopping power of an electron gas due to plasmon excitation from the rate of increase of energy in the plasmon field. An added feature of these calculations is that they bring together and illustrate subjects usually treated in different courses, namely, charged-particle penetration phenomena, as discussed in an electrodynamics course, with the coherent-states description as applied to the harmonic oscillator or the photon field in a quantum mechanics course.

## 2. Model boson system

The electron gas has been studied extensively for several decades and many modern solid-state physics textbooks include discussions of the electron-gas model for condensed media. A particularly complete description is given by Madelung (1978). It is known that many of its dynamic properties can be understood in terms of the plasmon pole approximation to its dielectric function (Hedin and Lundqvist 1969). In the present approach the plasmon is assumed to be of prime importance in determining the response of the electron gas to long wavelength disturbances. A simple model which is adequate to describe this important characteristic is the hydrodynamical representation of the motion of a charged fluid. In this section we use the description and notation of Ritchie and Wilems (1969). The density of electrons,  $n(\mathbf{r}, t)$ , their velocity  $\mathbf{v}(\mathbf{r}, t)$  and the scalar electric potential  $\phi(\mathbf{r}, t)$  due to departures from neutral equilibrium suffice to describe the perturbed motion of the 'electron fluid'. A uniform, positively charged background is assumed to be present to ensure overall charge neutrality. The equation of continuity

$$\nabla \cdot (\mathbf{v}n) = -\partial n / \partial t, \quad (1)$$

Poisson's equation

$$\nabla^2 \phi = 4\pi e(n - n_0) \quad (2)$$

and the Euler equation of motion

$$m \partial \mathbf{v} / \partial t + m(\mathbf{v} \cdot \nabla)\mathbf{v} = e\nabla\phi - m\beta^2 \nabla n / n_0 \quad (3)$$

are employed. Here  $n_0$  is the uniform, undisturbed density of electrons,  $m$  and  $e$  are electronic constants and  $\beta$  is the speed of propagation of a hydrodynamic disturbance in the system. The simple form of the last term in equation (3) is an approximation to the Euler pressure term  $n \int_0^n dp(n')/n'$  that is obtained by assuming that  $n$  varies relatively slowly with position. The pressure-density function (equation of state) of the fluid is  $p(n)$ .

In a conventional linearisation scheme, one sets  $n = n_0 + n_1 + n_2 + \dots$ ,  $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 + \dots$  and  $\phi = \phi_1 + \phi_2 + \dots$ , where the subscript denotes the order of 'smallness' of a given term in each of the

series. Substituting these expansions in equations (1)–(3) and neglecting terms of higher order than the first (subscript 1) and squares of first-order quantities, one comes to

$$n_0 \nabla \cdot \mathbf{v}_1 = -\partial n_1 / \partial t \quad (1a)$$

$$\nabla^2 \phi_1 = 4\pi e n_1 \quad (2a)$$

and

$$m \partial \mathbf{v}_1 / \partial t = e\nabla\phi_1 - m\beta^2 \nabla n_1 / n_0. \quad (3a)$$

It is straightforward to eliminate  $\mathbf{v}_1$  and  $\phi_1$  between these equations after taking the time derivative of equation (1a) and the divergence of equation (3a). One finds

$$(\partial^2 / \partial t^2 + \omega_p^2 - \beta^2 \nabla^2) n_1(\mathbf{r}, t) = 0 \quad (4)$$

for the electron density fluctuation function. We drop the subscripts on  $n_1$ ,  $\mathbf{v}_1$  and  $\phi_1$  henceforth. The plasma frequency is  $\omega_p = (4\pi n_0 e^2 / m)^{1/2}$ .

The energy residing in the electron fluid may be written

$$H = \frac{mn_0}{2} \int d^3r \mathbf{v}^2 + \frac{e^2}{2} \int d^3r \int d^3r' \frac{n(\mathbf{r}, t)n(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} + \frac{m\beta^2}{2n_0} \int d^3r n^2(\mathbf{r}, t). \quad (5)$$

The first term represents the kinetic energy, the second the Coulomb interaction between electron density fluctuations in different regions of space and the third represents the exchange-correlation energy. We expand  $n(\mathbf{r}, t)$  in a Fourier series of the form

$$n(\mathbf{r}, t) = \frac{1}{\Omega} \sum_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}) n_{\mathbf{k}}(t) \quad (6)$$

where  $\Omega$  is the normalisation volume. Expressing the velocity as the gradient of a scalar velocity potential  $\psi$ , i.e.  $\mathbf{v} = -\nabla\psi$ , one finds from equations (1a), (5) and (6)

$$H = \frac{m}{2n_0\Omega} \sum_{\mathbf{k}} \frac{1}{k^2} [\dot{n}_{\mathbf{k}}\dot{n}_{-\mathbf{k}} + (\omega_p^2 + \beta^2 k^2) n_{\mathbf{k}} n_{-\mathbf{k}}]. \quad (7)$$

Since  $n(\mathbf{r}, t)$  is real,  $n_{\mathbf{k}}$  and  $\dot{n}_{\mathbf{k}}$  are complex and satisfy the crossing relations  $n_{\mathbf{k}} = n_{-\mathbf{k}}^*$ ,  $\dot{n}_{\mathbf{k}} = \dot{n}_{-\mathbf{k}}^*$ . Thus we may write

$$H = \frac{m}{2n_0\Omega} \sum_{\mathbf{k}} \frac{1}{k^2} (\dot{n}_{\mathbf{k}}^* \dot{n}_{\mathbf{k}} + \omega_k^2 n_{\mathbf{k}}^* n_{\mathbf{k}}) \quad (8)$$

where  $\omega_k = (\omega_p^2 + \beta^2 k^2)^{1/2}$  is the frequency of a plasmon with wavenumber  $k$ . The Hamiltonian may be put into canonical form by the substitutions

$$n_{\mathbf{k}} = \frac{\mu_{\mathbf{k}}}{2\omega_{\mathbf{k}}} (a_{\mathbf{k}} + a_{-\mathbf{k}}^*) \quad \dot{n}_{\mathbf{k}} = \frac{-i\mu_{\mathbf{k}}}{2} (a_{\mathbf{k}} - a_{-\mathbf{k}}^*)$$

where  $\mu_{\mathbf{k}} = (2\hbar\omega_{\mathbf{k}}n_0\Omega k^2/m)^{1/2}$ . Then

$$H = \frac{1}{2} \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} (a_{\mathbf{k}}^* a_{\mathbf{k}} + a_{\mathbf{k}} a_{\mathbf{k}}^*) \quad (9)$$

which may be quantised by requiring that  $a_{\mathbf{k}}$  and  $a_{\mathbf{k}}^*$  be Heisenberg dynamical operators which obey boson commutation relations  $\{a_{\mathbf{k}}, a_{\mathbf{k}'}^{\dagger}\} = \delta_{\mathbf{k}, \mathbf{k}'}$  and  $\dot{a}_{\mathbf{k}} + i\omega_{\mathbf{k}} a_{\mathbf{k}} = 0$ . The Hamiltonian then reads

$$H = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} (a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \frac{1}{2}). \quad (10)$$

This model can be made to apply to a rather wider range of disturbances in an electron gas by taking  $\omega_{\mathbf{k}} = (\omega_p^2 + \beta^2 k^2 + \hbar^2 k^4 / 4m^2)^{1/2}$ . The addition of the last term in the brackets amounts to including the effects of single-particle recoil in the response of the medium and is equivalent to the 'plasmon pole approximation' that has been widely used in modelling the dynamic characteristics of the electron gas (Hedin and Lundqvist 1969).

The scalar electric potential  $\phi(\mathbf{r}, t)$  associated with the electron density  $n(\mathbf{r}, t)$  is

$$\phi(\mathbf{r}, t) = -e \int d^3 r' \frac{n(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|}. \quad (11)$$

Expressing  $n(\mathbf{r}, t)$  in terms of creation and annihilation operators, we find the operator corresponding to the scalar electric potential in the form

$$\phi(\mathbf{r}, t) = - \sum_{\mathbf{k}} g_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}) (a_{\mathbf{k}} + a_{-\mathbf{k}}^{\dagger}) \quad (12)$$

where

$$g_{\mathbf{k}} \equiv (2\pi \hbar \omega_p^2 / \omega_{\mathbf{k}} k^2 \Omega)^{1/2}. \quad (13)$$

Consider a charged particle penetrating the medium. The charge density corresponding to this particle may be written

$$\rho_{\text{ext}}(\mathbf{r}, t) = Ze \delta^3(\mathbf{r} - \mathbf{v}t) \quad (14)$$

if its charge is  $Ze$  and if it proceeds through the medium with constant velocity  $\mathbf{v}$ . The energy of interaction  $H'$  between this charged particle and electron density fluctuations in the medium may be written

$$\begin{aligned} H'(t) &= \int d^3 r \phi(\mathbf{r}, t) \rho_{\text{ext}}(\mathbf{r}, t) \\ &= -Ze \sum_{\mathbf{k}} g_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{v}t) (a_{\mathbf{k}} + a_{-\mathbf{k}}^{\dagger}). \end{aligned} \quad (15)$$

### 3. Wake potential

In the interaction picture (see e.g. Merzbacher 1961) the system wavefunction  $|\Psi_I(t)\rangle$  obeys the Schrödinger equation

$$i\hbar \frac{d}{dt} |\Psi_I(t)\rangle = H_I(t) |\Psi_I(t)\rangle \quad (16)$$

where

$$\begin{aligned} H_I(t) &= - \sum_{\mathbf{k}} Z e g_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{v}t) \\ &\quad \times [a_{\mathbf{k}} \exp(-i\omega_{\mathbf{k}}t) + a_{-\mathbf{k}}^{\dagger} \exp(i\omega_{\mathbf{k}}t)]. \end{aligned} \quad (17)$$

We assume that the system is in its unperturbed ground state at  $t = -\infty$ , so that  $|\Psi_I(-\infty)\rangle = |0\rangle$ . Let us introduce

$$\begin{aligned} \alpha_{\mathbf{k}}(t) &= \frac{iZ e g_{\mathbf{k}}}{\hbar} \int_{-\infty}^t ds \exp[-i(\mathbf{k} \cdot \mathbf{v} - \omega_{\mathbf{k}} + i\eta)s] \\ &= - \frac{Z e g_{\mathbf{k}} \exp[-i(\mathbf{k} \cdot \mathbf{v} - \omega_{\mathbf{k}} + i\eta)t]}{\hbar (\mathbf{k} \cdot \mathbf{v} - \omega_{\mathbf{k}} + i\eta)} \end{aligned} \quad (18)$$

where  $\eta$  is a positive infinitesimal. Then, equation (16) becomes

$$\frac{d}{dt} |\Psi_I(t)\rangle = \sum_{\mathbf{k}} \left( a_{\mathbf{k}}^{\dagger} \frac{d\alpha_{\mathbf{k}}(t)}{dt} - a_{\mathbf{k}} \frac{d\alpha_{\mathbf{k}}^*(t)}{dt} \right) |\Psi_I(t)\rangle. \quad (19)$$

The different modes are independent and  $|\Psi_I(t)\rangle = \prod_{\mathbf{k}} |\Psi_I^{\mathbf{k}}(t)\rangle$ . Thus we can solve for the evolution of each mode separately from the equation

$$\frac{d}{dt} |\Psi_I^{\mathbf{k}}(t)\rangle = \left( a_{\mathbf{k}}^{\dagger} \frac{d\alpha_{\mathbf{k}}(t)}{dt} - a_{\mathbf{k}} \frac{d\alpha_{\mathbf{k}}^*(t)}{dt} \right) |\Psi_I^{\mathbf{k}}(t)\rangle. \quad (20)$$

The solution to this equation is

$$|\Psi_I^{\mathbf{k}}(t)\rangle = D_{\mathbf{k}}(t) |\Psi_I^{\mathbf{k}}(-\infty)\rangle \quad (21)$$

where the unitary operator  $D_{\mathbf{k}}$  is defined by

$$D_{\mathbf{k}}(t) \equiv \exp(\alpha_{\mathbf{k}}(t) a_{\mathbf{k}}^{\dagger} - \alpha_{\mathbf{k}}^*(t) a_{\mathbf{k}}). \quad (22)$$

Using the Baker-Hausdorff identity (see e.g. Messiah 1964) which states that for operators  $A$  and  $B$ ,  $\exp(A)\exp(B) = \exp(A+B + \frac{1}{2}\{A, B\})$  if  $\{A, \{A, B\}\} = 0 = \{B, \{A, B\}\}$ , equation (22) may be expressed in the equivalent form

$$D_{\mathbf{k}}(t) = \exp(-\frac{1}{2} |\alpha_{\mathbf{k}}(t)|^2) \exp(\alpha_{\mathbf{k}}(t) a_{\mathbf{k}}^{\dagger}) \exp(-\alpha_{\mathbf{k}}^*(t) a_{\mathbf{k}}). \quad (23)$$

Then the state of the interacting system at time  $t$  is

$$|\Psi_I(t)\rangle = \prod_{\mathbf{k}} |\Psi_I^{\mathbf{k}}(t)\rangle = \prod_{\mathbf{k}} D_{\mathbf{k}}(t) |0\rangle. \quad (24)$$

The set of states  $|\Psi_I^{\mathbf{k}}(t)\rangle$  given by equation (21) are the coherent states describing the system of incident charged particles and plasmons. They are eigenvectors of the annihilation operator with complex eigenvalues, i.e.

$$a_{\mathbf{k}} |\Psi_I^{\mathbf{k}}(t)\rangle = \alpha_{\mathbf{k}}(t) |\Psi_I^{\mathbf{k}}(t)\rangle. \quad (25)$$

The coherent-state description of a system has the very important property that quantities having a classical analogue, such as the electric and magnetic fields for a system of photons, are predicted correctly. An elementary description of the coherent states for a forced harmonic oscillator has been given by Carruthers and Nieto (1965).

Equation (24) shows that the plasmon field is

excited to a set of coherent states due to the influence of the swift ion. We can now demonstrate that this formulation leads to the same expression for the wake potential as determined earlier using a different theoretical approach (Echenique *et al* 1979). For this purpose the wavefunction for the system can be written, using equations (24) and (23), as

$$|\Psi_I(t)\rangle = \prod_k A_k(t) \exp(\alpha_k(t) a_k^\dagger) |0\rangle \quad (26)$$

where  $A_k(t) \equiv \exp(-\frac{1}{2} |\alpha_k(t)|^2)$  and we assume  $a_k^n |\Psi_I^k(-\infty)\rangle = 0$  for  $n \geq 1$ , i.e. no plasmons are present at  $t = -\infty$ .

The wake potential,  $\Phi_w(\mathbf{r}, t)$ , is the mean value of the scalar potential operator. Thus

$$\Phi_w(\mathbf{r}, t) = \langle \Psi_I(t) | \phi_I(\mathbf{r}, t) | \Psi_I(t) \rangle \quad (27)$$

and from equations (12) and (26)

$$\begin{aligned} \Phi_w(\mathbf{r}, t) &= \sum_q g_q \langle 0 | \prod_{k'} A_{k'}(t) \\ &\quad \times \exp(\alpha_{k'}^* a_{k'}) \exp(i\mathbf{q} \cdot \mathbf{r}) \\ &\quad \times [a_q \exp(-i\omega_q t) + a_{-q}^\dagger \exp(i\omega_q t)] \\ &\quad \times \prod_k A_k(t) \exp(\alpha_k a_k^\dagger) |0\rangle. \end{aligned} \quad (28)$$

After some simple algebra, one obtains

$$\begin{aligned} \Phi_w(\mathbf{r}, t) &= -2 \operatorname{Re} \sum_k \alpha_k g_k \\ &\quad \times \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega_k t)] \\ &= 2 \operatorname{Re} \sum_k \frac{Z e g_k^2 \exp[i\mathbf{k} \cdot (\mathbf{r} - \mathbf{v}t)]}{\hbar \mathbf{k} \cdot \mathbf{v} - \omega_k + i\eta} \end{aligned} \quad (29)$$

where  $\operatorname{Re}(\ )$  denotes the 'real part of ( )'. If we let  $\Omega \rightarrow \infty$  so that  $\sum_k \rightarrow [\Omega/(2\pi)^3] \int d^3k$ , then

$$\Phi_w(\mathbf{r}, t) = \frac{Z e \omega_p^2}{2\pi^2} \operatorname{Re} \int \frac{d^3k \exp[i\mathbf{k} \cdot (\mathbf{r} - \mathbf{v}t)]}{k^2 \omega_k \mathbf{k} \cdot \mathbf{v} - \omega_k + i\eta}. \quad (30)$$

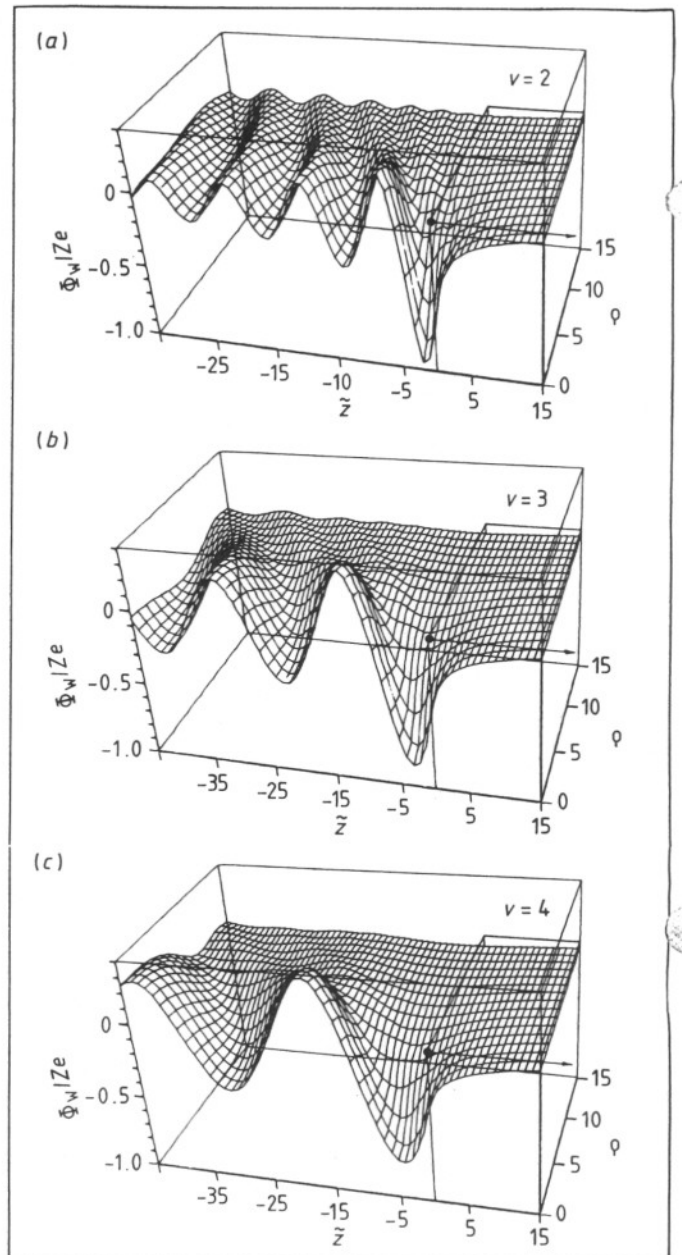
In cylindrical coordinates with the trajectory of the ion as the  $z$  axis and with  $\omega \equiv \mathbf{k} \cdot \mathbf{v} = k_z v$ , we obtain

$$\begin{aligned} \Phi_w(\mathbf{r}, t) &= \frac{Z e \omega_p^2}{\pi v} \int_0^\infty d\kappa \kappa J_0(\kappa \rho) \\ &\quad \times \int_{-\infty}^\infty \frac{d\omega}{\kappa^2 + (\omega/v)^2} \frac{\exp(i\omega \tilde{z}/v)}{\omega^2 - \omega_k^2 + 2i\eta\omega}. \end{aligned} \quad (31)$$

The cylindrical coordinates  $\rho$  and  $\tilde{z}$  are defined by  $\rho \equiv (x^2 + y^2)^{1/2}$  and  $\tilde{z} \equiv z - vt$  relative to the position  $(x, y, z) = (0, 0, vt)$  of the moving charge. The wave vector  $\mathbf{k}$  has component  $\kappa$  in the  $\rho$  direction and thus  $k = [\kappa^2 + \omega^2/v^2]^{1/2}$ . The speed of the moving charge is assumed to remain constant. If we set  $\omega_k = \omega_p$ , i.e. neglect dispersion in the plasmon energy, then equation (31) reproduces the well known result for the wake potential first derived using dielectric theory by Neufeld and Ritchie (1955a, b).

Using the plasmon pole approximation, as discussed below equation (10), equation (31) yields the same expression for  $\Phi_w$  as found by Echenique *et al* (1979) in the same approximation.

In figure 1 we display the wake potential  $\Phi_w(\rho, \tilde{z})$  (divided by  $Ze$ ) generated in the medium by the passage of a charged particle of speed  $v$  as calculated using the plasmon pole approximation. The wake potential is the homogeneous part of the total potential in the medium. All quantities are in atomic units (au) so that  $\Phi_w/Ze$  is in units of  $1/a_0$



**Figure 1** Wake potential surfaces  $\Phi_w(\rho, \tilde{z})/Ze$  as calculated for projectiles moving with speed  $v = (a) 2, (b) 3$  and  $(c) 4$  au through a medium characterised by  $\omega_p = 0.919$  au = 25 eV;  $\beta = 0.974$  au =  $2.13 \times 10^8$  m s $^{-1}$ . The particle is located at the origin of the  $(\tilde{z}, \rho, \Phi_w)$  coordinate system as indicated by the dot, and the arrow points in the direction of  $v$ .

while  $\rho$  and  $\bar{z}$  are in units of  $a_0$  where  $a_0 = 0.529 \text{ \AA}$  is the first Bohr radius. The potential energy of a test charge (+e) located at some point  $(\rho, \bar{z})$ , in eV, is  $U [\text{eV}] = 27.2 \Phi_w [\text{au}]$  while the kinetic energy in eV of a particle of mass  $M$  is related to the speed  $v$  in atomic units by  $E [\text{eV}] = 13.6(M/m)(v [\text{au}])^2$ . Note that in all three of these plots the potential  $\Phi_w$  does not include the bare (vacuum) potential of the charge but only the part of the potential generated by electrons moving in response to the charged particle. The wake potential is negative in the region immediately behind the particle because electrons tend to move so as to screen the charge. Since the charge is in motion, the screening electrons lag behind the charge and are set into oscillatory motion that, in the rest frame of the moving charge, gives rise to the spatial patterns shown. A more detailed description of these results was given by Echenique *et al* (1979). The structure of wakes is an important element in determining the response of a medium to a charged particle as alluded to in the introduction.

#### 4. Stopping power

We can make contact with a well known result for the stopping power of an electron gas by looking at the change with time of the energy in the plasmon field. The expectation value of  $H$ , equation (10), is

$$\langle \Psi_I(t) | H | \Psi_I(t) \rangle = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} (|\alpha_{\mathbf{k}}(t)|^2 + \frac{1}{2}). \quad (32)$$

The change of this quantity with time is found to be

$$\begin{aligned} \frac{d}{dt} \langle \Psi_I(t) | H | \Psi_I(t) \rangle &= \frac{4\pi Z^2 e^2 \omega_p^2}{\Omega} \\ &\times \sum_{\mathbf{k}} \frac{1}{k^2} \text{Im} \left( \frac{-1}{\mathbf{k} \cdot \mathbf{v} - \omega_{\mathbf{k}} + i\eta} \right) \end{aligned} \quad (33)$$

where  $\text{Im}(\ )$  indicates the 'imaginary part of ( )'. If we let  $\Omega \rightarrow \infty$  and use the identity

$$\lim_{\eta \rightarrow 0} \frac{1}{x + i\eta} = \text{P} \frac{1}{x} - i\pi \delta(x)$$

where P indicates the principal value integral is taken, equation (33) becomes

$$\frac{d}{dt} \langle \Psi_I(t) | H | \Psi_I(t) \rangle = \frac{Z^2 e^2 \omega_p^2}{2\pi} \int \frac{d^3 k}{k^2} \delta(\mathbf{k} \cdot \mathbf{v} - \omega_{\mathbf{k}}). \quad (34)$$

The energy gain per unit time in the plasmon field, shown by equation (34), is due to energy loss by the fast charged particle in its interaction with the medium. The energy loss of the particle per unit distance travelled,  $(1/v)(-dE/dt) \equiv -dE/dx$ , where

$-dE/dt$  is the energy loss of the particle per unit time, is the stopping power of the medium for the charged particle. Using a cylindrical coordinate system, we have

$$-\frac{dE}{dx} = \left( \frac{Ze\omega_p}{v} \right)^2 \int \frac{dk}{k} \int_{-1}^1 d\mu \delta\left(\mu - \frac{\omega_{\mathbf{k}}}{kv}\right). \quad (35)$$

If we take  $\omega_{\mathbf{k}} = \omega_p$  and restrict  $k$  to values smaller than some value  $k_c$  for which plasmons are reasonably well defined excitations in the medium, equation (35) yields the stopping power of the medium due to plasmon excitation as

$$\left( -\frac{dE}{dx} \right)_{\text{plasmon}} = \left( \frac{Ze\omega_p}{v} \right)^2 \ln(k_c v / \omega_p). \quad (36)$$

This simple result was discussed by Pines and Bohm (1952) and Pines (1953) in terms of the coherent emission of plasmons to form a wake trailing a fast ion moving through an electron gas.

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