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The wake of charged particles in condensed matter

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ABSTRACT

A brief discussion of the scalar electric potential generated by a swift charged particle in condensed matter is given. Collective modes give rise to an oscillatory wake stretching behind the particle, while electron–hole excitations are responsible for crispations that have the De Broglie wavelength of an electron travelling with the charged particle's velocity. Experimental manifestations of the wake are reviewed briefly. Completely quantal derivations of expressions for the wake potential are presented. The results are compared with the classically derived one.

§ 1. INTRODUCTION

Since the earliest nuclear researches of Rutherford and co-workers, swift charged particles have been used extensively to probe the structure of matter. Modern surface science, for example, owes much to information gained with such probes. Recent developments, discussed below, involve the use of correlated charged particles to study the spatial and temporal evolution of disturbances generated in condensed matter by the particles themselves. The theory of the charged particle wake has been discussed in the literature using classical electrodynamics. Here we apply quantal methods to analyse two important configurations and arrive at a description of the wake without invoking classical concepts.

§ 2. QUALITATIVE ASPECTS OF THE WAKE

A charged particle moving with velocity $\mathbf{v}$ in a medium having electron collective resonant frequency ω_0 causes electron-density fluctuations of two types. Collective motion behind the particle make up a Čerenkov-type pattern in a periodic wake of wavelength $2\pi v/\omega_0$. These fluctuations constitute a mode of energy transport from the particle track, leading finally to particle–hole excitations. Single-electron excitations give rise to bow waves of wavelength $\sim h/mv$ ahead of the particle and to periodic crispations in the collective portion of the wake. The electric field associated with the wake acts to retard the particle and can influence the motion of nearby particles.

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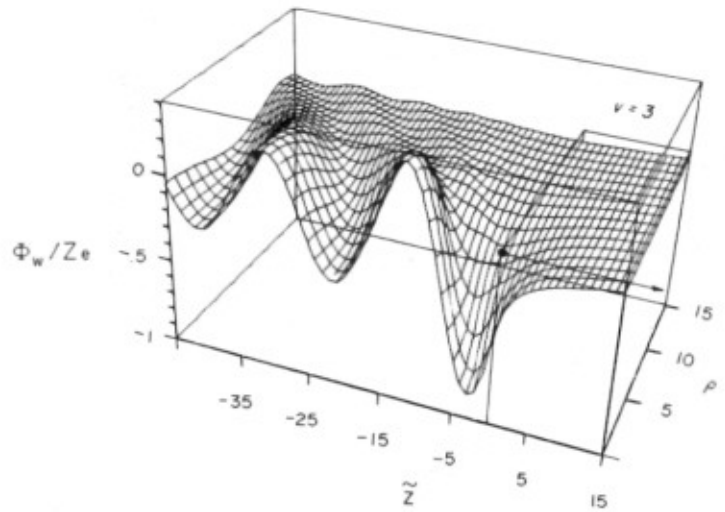
Wake theory has been discussed in the literature on the basis of classical electrodynamics (Vlasov 1950†, Neufeld and Ritchie 1955, Neelavathi, Ritchie and Brandt 1974, Ritchie, Brandt and Echenique 1976, Day 1975, Echenique 1977, Neelavathi and Kher 1976, Vager and Gemmell 1976, Day and Ebel 1979, Ritchie, Echenique, Brandt and Basbas 1979, Echenique, Ritchie and Brandt 1979, Day 1980). A swift particle with charge Ze is treated as a point moving with constant velocity $\mathbf{v}$ in a dielectric medium. The scalar electric potential generated is computed from Poisson's equation, given $\epsilon_{k,\omega}$, the dielectric response function of the medium. For many substances $\epsilon_{k,\omega}$ may be taken as a scalar to a good approximation. The resulting wake potential Φ_w may be written

$$\Phi_w(\mathbf{r}) = \frac{Ze}{2\pi^2} \int \frac{d^3k}{k^2} \exp(i\mathbf{k}\cdot\mathbf{r})(\epsilon^{-1}{}_{k,\mathbf{k}\cdot\mathbf{v}} - 1) \tag{1}$$

and is defined so that the potential of the bare charge is subtracted from the total scalar electric potential in the medium (Echenique *et al.* 1979). The origin of the coordinate $\mathbf{r}$ is taken at the charge and in its rest frame.

Fig. 1



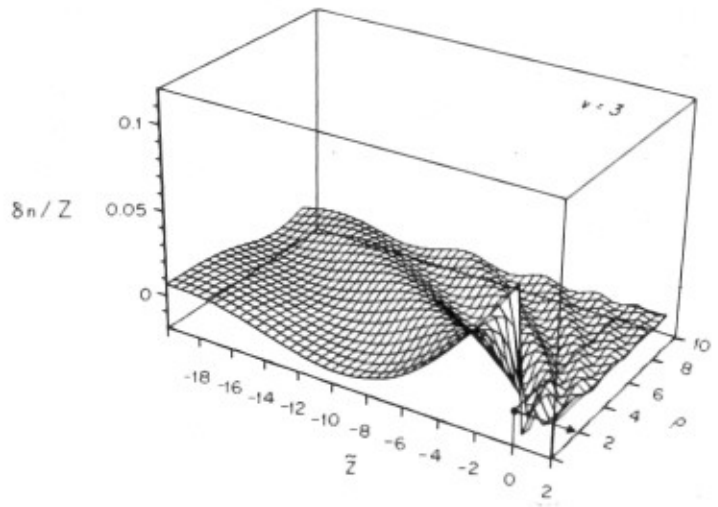
Wake potential surface $\Phi_w(\rho, \tilde{z})/Ze$ as calculated for a projectile moving with velocity $v=3$ a.u. in a medium characterized by a plasmon energy of 25 eV. The particle is located at the origin of the $(\rho, \tilde{z}, \Phi)$ coordinate system and the arrow points in the direction of $\mathbf{v}$.

Figure 1 shows $\Phi_w(\rho, \tilde{z})/Ze$ as it depends on ρ, the lateral distance from the charged particle track, and on $\tilde{z}$, the distance from the particle measured parallel to $\mathbf{v}$. The calculations leading up to this figure were made from eqn. (1)

† This book reviews the pioneering work of Vlasov in the theory of classical plasmas. It gives reference to his original papers in which the disturbance set up in a plasma by a test charge is studied.

using the dielectric function of an electron gas and for an ion with speed $v = 3e^2/\hbar = 6{\cdot}57 \times 10^8$ cm s^{-1} (Echenique *et al.* 1979). The density of the electron gas corresponded to $r_s = 1{\cdot}53$, or to a plasmon energy of 25 eV. Atomic units have been used. Thus the wavelength of the longest oscillatory portion is $\sim 20{\cdot}5$ a.u., or 10·8 Å, and the maximum depth of the first potential trough is $\sim 0{\cdot}7$ a.u., or 19 eV. The most prominent aspect of this distribution is the oscillatory character of that portion stretching behind the particle. One sees that Φ_w becomes quite small at lateral distances > 5 Å from the particle track. The oscillations of Φ_w in the z variable are seen increasingly to lag behind the particle as ρ increases, constituting a Čerenkov-like disturbance. Single-particle bow waves with wavelength $\sim h/mv$ are not readily seen ahead of the particle here, but show up vividly in representations of the electron density fluctuation function $\delta n(\rho, \tilde{z}) = \nabla^2 \Phi_w / 4\pi e$ (Echenique *et al.* 1979).

Fig. 2



Electron density fluctuation function $\delta n(\rho, \tilde{z})/Z$ as calculated for the same conditions as fig. 1.

Such a plot is shown in fig. 2 for the same ion velocity in the same medium, but to a somewhat different scale of ρ and $\tilde{z}$. Atomic units are also employed here and the scaled quantity $\delta n/Z$ is displayed. Multiple bow waves of intriguing complexity are clearly visible. These take the form of paraboloidal patterns with foci at the particle, extending behind it with an asymptotic half-angle of opening $\sim \sin^{-1}(v_F/v)$, where v_F is the Fermi speed of electrons in the medium. Such bow waves correspond in the forward hemisphere to knock-on electrons that have experienced small impact-parameter collisions with the ion and thereby acquire speeds $\sim v$. They merge continuously in backward directions with forms of larger wavelength that represent collective disturbances in the valence electron sea.

§ 3. Wake effects in ion penetration

Ion clusters are created by injecting swift molecular ions such as H_2^+ into a solid target. The ions are stripped of their electrons after penetrating only a few atomic layers. The initial separation of the constituent ions is ~ 1 Å. The wake generated by the resulting tight cluster is given by a superposition of wakes due to the individual ions. The dynamically modified Coulomb repulsion between its constituents causes the cluster to explode. If the target is thin enough the cluster will emerge with nearly its original energy and unmodified Coulomb repulsion will operate until the fragments are collected in detectors. For a target ~ 100 Å thick the dwell time of a cluster inside the target is $\sim 10^{-15}$ s in typical experiments. The time for the explosion to run its course, converting most of the initial electrostatic energy into kinetic energy of the separating fragments, is also $\sim 10^{-15}$ s.

The first experimental evidence of a wake effect in the energy loss of ion clusters was given by Brandt, Ratkowski and Ritchie (1974). Swift H_2^+ and H_3^+ ions bombarded thin foils. It was found that the energy lost by an ion cluster in a foil was larger than that which would have occurred if the constituent ions were isolated from one another. The augmented energy loss is due to the retarding effect of the wake of a leading ion on a trailing one. These authors analysed the cluster energy loss theoretically in the same paper, while Arista and Ponce (1975) and Arista (1978) gave independent analyses. Tape, Gibson, Remillieux, Laubert and Wegner (1976) made energy loss measurements of diclusters formed from O_2^+ ions, with similar results (see also Eckardt, Lantschner, Arista and Baragiola 1978).

An ion trailing another at fixed separation and at the same velocity will experience a force tending to move it toward the track of the first if it is in the region $-\pi v/\omega_0 < \tilde{z} < -v/\omega_0$. This force arises because there is an excess of polarized electrons behind the leading ion. The reality of this aligning force was demonstrated by Gemmell and co-workers (Gemmell, Remillieux, Gaillard, Holland and Vager 1975) in the angular distribution of protons emerging from Au single crystals bombarded by $(HeH)^+$ beams under planar channelling directions.

Observations of the explosion process have been made in a series of high-resolution studies of the distribution in energy and angle of fragments produced when a variety of fast diatomic molecular ions bombard thin polycrystalline foils (Golovchenko and Laegsgaard 1974, Gemmell *et al.* 1975, 1978, Vager and Gemmell 1976, Kanter, Cooney, Gemmell, Groeneveld, Pietsch, Ratkowski, Vager and Zabrnasky 1979). When H_2^+ ions are used, for example, a bimodal distribution in energy is found at a direction nearly parallel to that of the incident ions. An asymmetry observed corresponds to the presence of larger numbers of trailing protons than leading ones and is consistent with the presence of a wake that exerts a transverse force on trailing particles. Valuable information about the steric structure of several different polyatomic ions has been gained in experiments of this kind (Gaillard, Gemmell, Goldring, Levine, Pietsch, Poizat, Ratkowski, Remillieux, Vager and Zabransky 1978).

Recent experiments with swift channelled ions having atomic numbers ranging from 5 to 9 and that carry a single electron in a K-orbital state show resonant excitation of the electron due to coherent periodic perturbation by atoms in the bounding crystal rows (Datz, Moak, Crawford, Krause, Dittner,

del Campo, Biggerstaff, Miller, Hvelplund and Knudsen 1978, Moak, Datz, Crawford, Krause, Dittner, del Campo, Biggerstaff, Miller, Hvelplund and Knudsen 1979). A pronounced splitting in the resonant dips of the surviving fraction of these ions as a function of ion speed may be attributed in substantial portion to the wake of the ion (Crawford and Ritchie 1979).

§ 4. Quantal theory of the wake : the ion cluster

For simplicity we consider a diatomic molecular ion prepared in a well-defined state of rotational and vibrational motion and incident with velocity $\mathbf{v}$ on a solid target. The valence electrons are rapidly swept away leaving the dicluster in a wave packet that we describe by the initial state vector $\langle \mathbf{r}, \mathbf{R}|0\rangle = u_0(\mathbf{r}) \exp(i\mathbf{K}\cdot\mathbf{R})/\sqrt{\Omega}$. The internuclear coordinate is $\mathbf{r}$, the co-ordinate of the centre of mass is $\mathbf{R}$ and $\mathbf{K} = (M_\alpha + M_\beta)\mathbf{v}/\hbar$, where M_α and M_β are the masses of the constituent ions.

The second-order expression for the shift ΔE in the energy of the interacting system may be written as

$$\Delta E = \sum_n \sum_f \frac{|\langle\psi_n|\langle f|V|0\rangle|\psi_0\rangle|^2}{\xi_0 + \epsilon_0 - \xi_f - \epsilon_n + i\delta}, \tag{2}$$

where $|\psi_n\rangle$ and ϵ_n are the exact state vector and energy of the nth excited state of the medium, ξ_0 is the initial cluster energy and ξ_f is its energy in an arbitrary eigenstate. The infinitesimal δ is positive.

The Coulomb interaction energy

$$V = -Z_\alpha e^2 \sum_{i=1}^{N} \frac{1}{|\mathbf{r}_\alpha - \mathbf{r}_i|} - Z_\beta e^2 \sum_{i=1}^{N} \frac{1}{|\mathbf{r}_\beta - \mathbf{r}_i|} \tag{3}$$

between the nuclei at positions $\mathbf{r}_\alpha$ and $\mathbf{r}_\beta$ with charges $Z_\alpha e$ and $Z_\beta e$ and the N electrons in the medium does not include the Coulomb repulsive energy between the ions because the state $|f\rangle$ of the pair is taken to be an eigenfunction of Schrödinger's equation that includes this Coulomb energy. Taking $\langle \mathbf{r}, \mathbf{R}|f\rangle = \Phi_f(\mathbf{r}) \exp(i\mathbf{K}_f\cdot\mathbf{R})/\sqrt{\Omega}$ we obtain from eqns. (2) and (3)

$$\Delta E = \Omega^{-1} \sum_n \sum_{\mathbf{q}_f} \sum_{\mathbf{k}} v_k{}^2 \frac{|\langle\Phi_f|Z_\alpha \exp(i\mu_\beta\mathbf{k}\cdot\mathbf{r}) + Z_\beta \exp(i\mu_\alpha\mathbf{k}\cdot\mathbf{r})|u_0\rangle\rho_{n0}(\mathbf{k})|^2}{(\hbar\mathbf{k}\cdot\mathbf{v} - (\hbar^2k^2/2M) + \tilde{E}_0 - E_f + \epsilon_0 - \epsilon_n + i\delta)}, \tag{4}$$

where $\mathbf{k} = \mathbf{K} - \mathbf{K}_c$ is the change in centre-of-mass momentum in the virtual transition. Here $v_k = 4\pi e^2/k^2$ and

$$\rho_{n0}(\mathbf{k}) = \langle\psi_n| \sum_{i=1}^{N} \exp(-i\mathbf{k}\cdot\mathbf{r}_i)|\psi_0\rangle$$

is the matrix element of the density operator for electrons in the medium. $\tilde{E}_0$ is the energy of internal motion in the initial state, while E_f is the energy of internal motion corresponding to the repulsive Coulomb eigenfunction, Φ_f, with wave-vector $\mathbf{q}_f$. Also, $M = M_\alpha + M_\beta$ and $\mu_{\alpha,\beta} = M_{\alpha,\beta}/M$. We now argue that the initial state wavefunction $u_0(r)$ is expected to have a spatial extension corresponding to atomic dimensions, e.g. ~ 1 Å. Then its momentum transform should be very small for momenta $q_f \gg 1$ Å^{-1}. Because $E_f \approx \hbar^2 q_f{}^2/M$ in this regime, it should be an excellent approximation to neglect both E_f and the

term $\hbar^2k^2/2M$ in the denominator of eqn. (4) in comparison with the other terms in view of the largeness of M. This is equivalent to the statement that cluster explosion will occur slowly on a time-scale of $\sim 1/\omega_0$, the response time of electrons in the medium. It should then be a good approximation to compute the self-energy adiabatically at each stage in the expansion. Theoretical determination of this time evolution is non-trivial since it involves dynamical self-consistency requirements. Hence we proceed to derive an expression that should be quite accurate during early expansion. We invoke closure to carry out the sum over intermediate states of internal dicluster motion, neglecting E_f in the denominator of eqn. (4). This results in the expression

$$\Delta E = \Omega^{-2} \sum_{\mathbf{k}} v_k^2 \langle u_0 | | Z_\alpha \exp(-i\mu_\beta \mathbf{k}\cdot\mathbf{r}) + Z_\beta \exp(i\mu_\alpha \mathbf{k}\cdot\mathbf{r})|^2 | u_0\rangle \sum_n \frac{|\rho_{n0}(\mathbf{k})|^2}{\hbar\mathbf{v}\cdot\mathbf{k} + \bar{E}_0 + \epsilon_0 - \epsilon_n + i\delta}. \quad (5)$$

The sum involving n may be formally evaluated in terms of the retarded dielectric response function of an electron gas (Pines and Nozieres 1966). Thus

$$\Delta E = \int d^3r\ |u_0(\mathbf{r})|^2 \frac{e^2}{2\pi^2} \int \frac{d^3k}{k^2}\ |Z_\alpha + Z_\beta \exp(-i\mathbf{k}\cdot\mathbf{r})|^2 [\epsilon^{-1}{}_{\mathbf{k},\mathbf{k}\cdot\mathbf{v}+E_0} - 1]. \quad (6)$$

In eqn. (6) we have found a quantal expression equivalent to the classical eqn. (1) for the wake potential. Only the cross-terms obtained by expanding the absolute squared term in the second integral of eqn. (6) actually depend on r, the internuclear coordinate. These terms give rise to an energy shift that may be written as

$$\Delta E_w = \int d^3r\ |u_0(\mathbf{r})|^2 \frac{Z_\alpha Z_\beta e^2}{2\pi^2} \int \frac{d^3k}{k^2} \exp(i\mathbf{k}\cdot\mathbf{r}) [\epsilon^{-1}{}_{\mathbf{k},\mathbf{k}\cdot\mathbf{v}+E_0} - 1] \quad (7)$$

plus a term in which $\mathbf{r}$ is replaced by $-\mathbf{r}$ in the exponential. The integral over k in eqn. (7) represents the contribution of the wake of one ion evaluated at the position of the other and is averaged over the probability density $|u_0(\mathbf{r})|^2$ of finding the ions separated by the vector $\mathbf{r}$. It is clear that the k integral of eqn. (7) is very similar to eqn. (1) and represents a quantal generalization of it. The energy shift $\Delta E(t)$ at a time t in the expansion of the dicluster may be computed in the present adiabatic, Born–Oppenheimer-type approximation by replacing $|u_0(r)|^2$ by the probability density

$$|\psi(\mathbf{r}, t)|^2 = \left| \sum_f \langle \chi | u_0 \rangle \exp(-i\omega_f t) \chi_f(\mathbf{r}) \right|^2.$$

Here ω_f and $\chi_f(\mathbf{r})$ are the energy and eigenfunction of the fth state of the dicluster's internal motion. These eigenfunctions should be determined self-consistently with the dynamical response of the medium. At a sufficiently early stage in the expansion the Coulombic eigenfunction set may be used. The energy shift of the dicluster interacting with the medium may be averaged over the part of the expansion appropriate in a given experiment.

§ 5. QUANTAL THEORY OF THE WAKE : THE ION-BOUND-ELECTRON SYSTEM

In terms of destruction operators $C_{\mathbf{K}n}$ for a state in which an ion-bound-electron complex has centre-of-mass momentum $\mathbf{K}$ and internal motion specified by the multicomponent quantum index n, the energy of the complex may be written as

$$H_0 = \sum_{\mathbf{K}} \sum_{n} E_{\mathbf{K}n} C_{\mathbf{K}n}^{\dagger} C_{\mathbf{K}n}, \tag{8}$$

where $E_{\mathbf{K}n} = \mathbf{K}^2/2(M+1) + \omega_n$, ω_n is the energy of internal motion and M the ion mass. In this section we will use atomic units ($e = \hbar = m = 1$). Similarly, the kinetic energy operator of a free-electron gas is

$$H_e = \sum_{\mathbf{p}} \epsilon_p a_{\mathbf{p}}^{\dagger} a_{\mathbf{p}}, \tag{9}$$

where $a_{\mathbf{p}}$ is a destruction operator for an electron with momentum $\mathbf{p}$, and $\epsilon_p = \mathbf{p}^2/2$.

The energy of interaction between the ion–electron system and the electron gas may be written as

$$H_1 = \Omega^{-1} \sum_{n} \sum_{n'} \sum_{\mathbf{K}} \sum_{\mathbf{k}} \sum_{\mathbf{p}} v_k [\rho_{n'n}(\mathbf{k}) - Z\delta_n{}^{n'}] C^{\dagger}{}_{\mathbf{K}-\mathbf{k},n'} C_{\mathbf{K}n} a^{\dagger}{}_{\mathbf{p}+\mathbf{k}} a_{\mathbf{p}}, \tag{10}$$

where Ω is the normalization volume, $\rho_{n'n}(\mathbf{k}) = \langle n' | \exp(-i\mathbf{k} \cdot \mathbf{r}) | n \rangle$, $\mathbf{r}$ is the coordinate of the electron measured from the ion and Z is the charge of the ion. The Dyson equation for $G_{\mathbf{K}n,\omega}$, the exact Green function of the interacting system, may be solved formally as

$$G_{\mathbf{K}n,\omega} = [\omega - (E_{\mathbf{K}n} + \Sigma_{\mathbf{K}n,\omega})]^{-1}, \tag{11}$$

where $\Sigma_{\mathbf{K}n,\omega}$ is the self-energy. The latter may be represented by an infinite sum of proper self-energy diagrams in which interactions with the medium can occur in all possible ways. The energy and the damping rate of the ion–electron complex are found from the poles of the Green function. Thus, from eqn. (11) a transcendental equation for the complex energy, ω, is found :

$$\omega = E_{\mathbf{K}n} + \Sigma_{\mathbf{K}n,\omega},$$

which may be solved in first order by setting $\Sigma_{\mathbf{K}n,\omega} \approx \Sigma_{\mathbf{K}n,E_{\mathbf{K}n}}$. A manageable approximation to Σ may be had in the pair approximation (Schultz 1964). One obtains

$$\Sigma_{\mathbf{K}n,E_{\mathbf{K}n}} = \sum_{n'} \int \frac{d^3k}{(2\pi)^3} |\rho_{n'n}(\mathbf{k}) - Z\delta_n{}^{n'}|^2 v_k [\epsilon^{-1}{}_{\mathbf{k},\mathbf{v}\cdot\mathbf{k}+\omega_n-\omega_{n'}} - 1] \tag{12}$$

where $\mathbf{v} \equiv \mathbf{K}/(M+1)$ is the velocity of the centre-of-mass. It has been assumed that $M \gg 1$ in arriving at eqns (10) and (12).

From the structure of eqn. (12) it may be seen that distinct contributions to Σ originate from (a) transitions of the projectile in which its internal state remains unaltered and (b) transitions in whicht he internal state changes. Consider the four terms of eqn. (12) that appear when the absolute square bracket is multiplied out. The term proportional to Z^2 arises from the reaction of the ion on itself through polarization induced in the medium(it is of no interest here and will not be considered further). The two terms proportional to Z may be considered to originate (a) in the action of the ion on the electron

and (*b*) in the action of the electron on the ion. The second of these may also be disregarded for present purposes. The remaining portion $\Sigma_n{}^e$ of the self-energy is written as

$$\Sigma_n{}^e = \sum_{n'} \int \frac{d^3k}{(2\pi)^3} v_k \int d^3r\, u_{n'}{}^*(\mathbf{r}) \exp(i\mathbf{k}\cdot\mathbf{r}) u_n(\mathbf{r}) [\rho_{nn'}{}^*(\mathbf{k}) - Z\delta_n{}^{n'}][\epsilon^{-1}{}_{k,\mathbf{v}\cdot\mathbf{k}+\omega_n-\omega_{n'}} - 1], \quad (13)$$

where $u_n(\mathbf{r}) \equiv \langle \mathbf{r}|n\rangle$ and $\delta_n{}^{n'}$ is the Kronecker delta.

An expression for the self-energy as it depends on the coordinate $\mathbf{r}$ may be found directly by the argument that $\Sigma_n{}^e$ in eqn. (13) must be the expectation value of $\Sigma_n{}^e(\mathbf{r})$ with respect to the wavefunction of relative motion in the nth state, i.e.

$$\Sigma_n{}^e = \int d^3r\, u_n{}^*(\mathbf{r}) \Sigma_n{}^e(\mathbf{r}) u_n(\mathbf{r}). \quad (14)$$

Equating the integrands of eqns (13) and (14), it follows that

$$\Sigma_n{}^e(\mathbf{r}) = \sum_{n'} \int \frac{d^3k}{(2\pi)^3} v_k \frac{u_{n'}{}^*(\mathbf{r})}{u_n{}^*(\mathbf{r})} \exp(i\mathbf{k}\cdot\mathbf{r}) [\rho_{nn'}{}^*(\mathbf{r}) - Z\delta_n{}^{n'}][\epsilon^{-1}{}_{\mathbf{k},\mathbf{k}\cdot\mathbf{v}+\omega_n-\omega_{n'}} - 1]. \quad (15)$$

The term in eqn. (15) that is proportional to Z,

$$\Sigma_{Zw}(\mathbf{r}) = -Z \int \frac{d^3k}{(2\pi)^3} v_k \exp(i\mathbf{k}\cdot\mathbf{r}) [\epsilon^{-1}{}_{\mathbf{k},\mathbf{k}\cdot\mathbf{v}} - 1], \quad (16)$$

may be seen to be essentially the same as the semiclassical wake potential Φ_w of eqn. (1), i.e. the real part of Σ_{Zw} is the standard wake potential expressed in atomic units and multiplied by the electron charge (-1).

The part of eqn. (15) containing the factor $\rho_{nn'}{}^*(k)$ arises from the action of the electron's wake on the electron itself. The set of single-particle eigenfunctions $\{\langle \mathbf{r}|n\rangle\}$ that determines the matrix elements $\rho_{n'n}$ should be solved self-consistently, given that the self-energy enters the Schrödinger equation for $\{\langle \mathbf{r}|n\rangle\}$. An exact solution cannot be obtained. Useful approximations valid for eigenvalue spectra closely spaced compared with ω_0, or alternatively, widely spaced compared with ω_0, may be obtained by invoking closure to evaluate the sum over n' in eqn. (15) or, respectively, to neglect all terms except for $n' = n$ (Ritchie and Echenique 1981). Work towards obtaining self-consistent wavefunctions for internal motion will be reported elsewhere.

The approach described above may be used to define the wake potential generated by a projectile consisting of bound particles having arbitrary masses, e.g. a swift positronium atom (Ritchie and Echenique 1981).

This technique has been applied to analyse wake-splitting and lifetime broadening of hydrogenic electron states on projectiles with high atomic numbers (Ritchie and Echenique 1981, Crawford and Ritchie 1979, Tejada, Echenique, Crawford and Ritchie 1980). Reasonable agreement with experiment (Datz *et al.* 1978, Bell, Betz, Panke and Stehling 1976, Bell, Betz, Panke, Stehling and Spindler 1976) is obtained. Differences that are found are expected to disappear with the use of improved dielectric functions.

§ 6. Summary

We have given a brief review of some aspects of charged particle wake phenomena. Using quantal methods we have analysed two configurations of

importance in experiment : (1) the interaction of swift ion diclusters undergoing Coulomb explosion in a dielectric medium, and (2) the interaction with an electron gas of a swift ion-bound-electron projectile. We find quantal generalizations of the classical expression for the wake potential that are appropriate to these configurations. Systematic analysis of detailed experimental data obtained with molecular ion projectiles incident on thin foils promises to yield important information about the dynamical response of condensed matter.

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