

The existence and detection of Rydberg states at surfaces

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Received 15 December 1977

Abstract. It is shown that the surface barrier potential can confine electrons in surface states, which because of the coulombic tail on the potential form a Rydberg series. This previously known result is made more rigorous by a discussion of lifetime broadening of the states. Observability of the states via LEED experiments is also investigated. The Rydberg series should give rise to structure in LEED curves which can in principle be resolved for all members of the series as $n \rightarrow \infty$.

1. Introduction

It is well known that important elements in the electronic properties of surfaces are the bound states of the surface, so called surface states, localised in the direction normal to the surface. An accepted ingredient of semiconductor surfaces for this sort of state has now been observed on transition metals and nearly-free-electron metals. The surface states we discuss in this paper have a somewhat different origin from those most commonly encountered and to illustrate this we adopt the approach to surface state calculations taken by Pendry and Gurman (1975).

They viewed a surface state as a wave trapped between the bulk crystal and the surface barrier (figure 1). If a wave ψ_+ , carries unit flux towards the crystal a portion will be Bragg reflected

$$r_C e^{i\phi_C} \psi_- \quad (1)$$

where ψ_- carries unit flux away from the crystal towards the surface barrier. In its turn ψ_- will impinge on the surface barrier and be reflected back towards the crystal.

$$r_B e^{i\phi_B} r_C e^{i\phi_C} \psi_+ \quad (2)$$

Summing the repeated scatterings gives for the total amplitude of ψ_+ :

$$[1 - r_B r_C \exp i(\phi_B + \phi_C)]^{-1} \quad (3)$$

A pole in expression (3) denotes a bound state of the surface—i.e. a surface state. The waves ψ_+ and ψ_- are in general Bloch waves but in this paper it will be assumed that they are plane waves, for simplicity. The complications of coupling together more than two waves will be dealt with in a later section.

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Thus our conditions for a surface state are

$$r_B r_C = 1, \quad (4)$$

$$\phi_B + \phi_C = 2\pi n. \quad (5)$$

Since we know from flux conservation that

$$r_B \leq 1, \quad r_C \leq 1, \quad (6)$$

equation (4) implies that both r_B and r_C are unity. This means that the energy (E) and

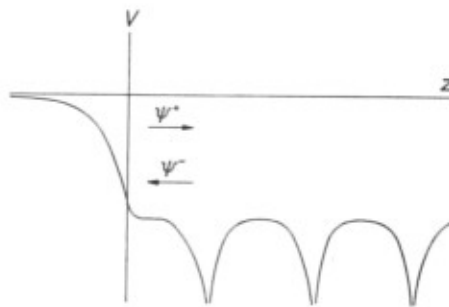


Figure 1. Schematic potential at the crystal surface showing atomic potentials and the surface barrier. A surface state is created by multiple reflections between the atomic potentials and the surface barrier.

momentum parallel to the surface (k_{\parallel}) of the waves must be such that there are no propagating states within the crystal and no possibility of flux escaping from the crystal into vacuum (conservation of momentum parallel to the surface may still prevent escape even if the energy is above the vacuum zero). This still leaves condition (5) on the phases to be satisfied and in general this will not happen for an arbitrary E and k_{\parallel} . However, the phases are functions of energy and in particular, Pendry and Gurman (1975) showed that when the energy sweeps across a band gap of the crystal ϕ_C will vary by π giving a good chance that somewhere in the gap a surface state will be found. We stress the point that variation of either ϕ_C or ϕ_B is an important element in producing a surface state: the greater the variation the greater the chance of sweeping through the 2π condition. Thus we shall refer to a surface state induced primarily by a rapid variation of ϕ_C as a crystal induced state, and that induced by a rapid variation of ϕ_B as a barrier induced state. Most states studied so far, such as the 'dangling bond' surface states on silicon are crystal-induced states, being intimately connected with the electronic properties of the bulk crystal. In contrast, in this paper we shall be concerned with the barrier induced states.

At first sight the division may appear to have been made on purely technical grounds, but we can show that there is a deeper significance to our assignment. Consider the case of reflection of a wave from a boundary. Let us assume that the condition of total reflectivity is fulfilled. Now even though completely reflected, the wave does extend to the far side of the boundary as an evanescent wave, and we can find an expression for the amount of charge on the far side of the boundary by introducing an infinitesimal constant imaginary component to the potential, $-i\epsilon$. From the Schrödinger equation it

then follows that

$$\epsilon \int_+ |\psi|^2 d^3r = \frac{-1}{2i} \int (\psi^* \nabla \psi - \psi \nabla \psi^*) \cdot dS \quad (7)$$

where we are working in atomic units such that $\hbar = e = m = 1$.

The volume integral on the left is over the space on the far side of the boundary and the surface integral on the right is over the boundary itself and represents flux flowing into the far side of the boundary. An alternative expression for this flux is to be had from the energy dependence of the reflectivity (r_c is unity)

$$\begin{aligned} \exp i \phi_c(E + i\epsilon) &= \exp i \left(\phi_c(E) + i\epsilon \frac{\partial \phi_c}{\partial E} \right) \\ &= \left(1 - \epsilon \frac{\partial \phi_c}{\partial E} \right) \exp(i\phi_c) \end{aligned} \quad (8)$$

and since unit flux was incident we can equate charge on the crystal side of the boundary from equation (7)

$$k \epsilon \frac{\partial \phi_c}{\partial E} = \epsilon \int_+ |\psi|^2 d^3r. \quad (9)$$

A similar equation gives us the charge on the vacuum side of the boundary

$$k \epsilon \frac{\partial \phi_B}{\partial E} = \int_- |\psi|^2 d^3r. \quad (10)$$

Thus our barrier induced surface state is defined to have a rapid variation of ϕ_B , also by virtue of (10) it has most of the charge on the vacuum side of the surface, see figure 2.

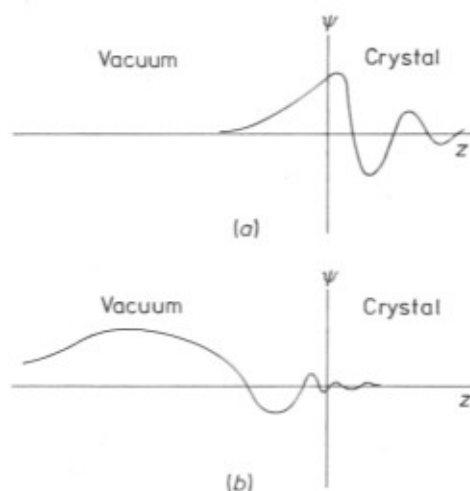


Figure 2. (a) Schematic wavefunction for a crystal induced surface state with energy well below vacuum zero. (b) Schematic wavefunction for a barrier induced surface state. Note the concentration of amplitude far from the surface and the small amplitude at the surface itself, typical of a higher member of the Rydberg-series.

Equations (9) and (10) are qualitatively understood from a WKB standpoint: a rapid change of phase with energy implies that the wavefunction must travel a long way on the far side of the boundary before it reaches the turning point, thus accumulating a large

phase. It also follows from this argument that large values of $\partial\phi_B/\partial E$ can only be expected at values of E and k_{\parallel} such that the electron can almost escape from the crystal: in that case the electron travels far out into the long coulombic tail of the surface barrier. In fact as threshold is approached, the variation of ϕ_B becomes infinitely rapid and a Rydberg series of states is generated. Barrier induced states are almost always part of a Rydberg series.

Hydrogenic states of this kind were first suggested on theoretical grounds by Cole and Cohen (1969) who noted that liquid helium showed total reflectivity to electrons near the vacuum zero in energy, and therefore should trap electrons in Rydberg states. The insulating character of helium reduces the probability of loss from these states and makes their observation by conductivity techniques feasible.

In this paper we take a more general viewpoint and examine the possibility that these states exist in metallic systems possibly at energies well above the vacuum zero where only momentum conservation prevents escape. The difficulty in metals, and in insulators too when the electron energy is high enough, is that electron-electron interaction results in decay processes that broaden surface states to resonances and this broadening may be so severe as to destroy all but the lowest member of the Rydberg series.

We expect the effect of inelastic scattering to be particularly important when the energy associated with the electron's movement parallel to the surface is bigger than the energy of the surface modes of the system. As the energy normal to the surface approaches the vacuum level, the states get closer in energy (in E_{\perp}), but at the same time the centre of gravity of the resonant state charge distribution moves further away from the surface, hence, having less probability of coupling to surface excitations, the energy broadening of the levels becomes smaller. We shall show that these two effects compensate in such a way that allows resonant levels to be resolved.

The observation of barrier induced surface states is probably most readily done by means of a low energy electron diffraction (LEED) experiment. At energies above the vacuum zero the surface state always has some coupling to beams that can escape from the surface and causes characteristic peaks and minima in the reflectivities as was shown by McRae (1971). In fact, low energy electron diffraction experiments have clearly demonstrated the reality and existence of resonant surface states at energies in the 0–100 eV range. (McRae and Caldwell 1967, Anderson 1970, Lauzier *et al* 1971, McRae and Wheatley 1972, Sinaroy *et al* 1972, McRae and Caldwell 1976a,b.)

Several studies of surface resonance effects by LEED at very low energy have also been made. (Jennings 1971, Jennings and Read 1974, Garcia and Solana 1976.) An analysis of inelastic scattering effects on the Rydberg spectrum has been made by Echenique (1976) and recently Rundgren and Malmström (1977) have made calculations of the effect of such surface states on LEED spectra. A recent experiment by McRae and co-workers seems to confirm Rundgren and Malmström's prediction (McRae *et al* 1977).

In this paper we first give a simple qualitative argument that shows why a Rydberg series of barrier induced surface states should exist. We go on to make this argument more rigorous, but stay within an analytic approach to the problem, and finally we discuss the possibilities for observation of these states through a LEED experiment.

2. Integrity of the Rydberg series—qualitative arguments

If the waves incident on and reflected from the surface barrier are plane waves, we can

write the total energy as

$$E = \frac{1}{2}|\mathbf{k}_{\parallel}|^2 + E_{\perp}. \quad (11)$$

If the asymptotic part of the surface barrier potential were continued in perfectly coulombic form up to some point $z = 0$:

$$\begin{aligned} V_B(z) &= -Z/|z|, & z < 0 \\ &= +\infty, & z > 0 \end{aligned} \quad (12)$$

then E_{\perp} would have the level structure

$$E_{\perp} = -\frac{1}{2} \frac{Z^2}{n^2}. \quad (13)$$

For a metal, Z takes the value of $\frac{1}{4}$ and for insulators it will be less:

$$Z = \frac{1}{4} \frac{\epsilon - 1}{\epsilon + 1} \quad (14)$$

where ϵ is the dielectric constant. So we are dealing with a maximum binding energy of around $\frac{1}{32}$ of a Hartree (~ 1 eV). The level spacing is

$$\Delta E_{\perp} = \frac{Z^2}{n^3} \quad (15)$$

and if lifetime effects broaden the state more than this it will not be possible to resolve the series.

Inside the crystal at the sort of energies used in LEED experiments, the broadening by lifetime effects is represented by an imaginary component to the self energy, Σ_i which takes a value between -0.04 and -0.05 H. Clearly if these values held for the surface states, no Rydberg series would be possible! What saves the day is the location of the centre of gravity of the surface state outside the crystal, as we discussed in the Introduction, because the decay processes responsible for the lifetime fall rapidly in strength outside the surface as shown by Inkson (1973) and Echenique and Pendry (1975). Beyond a certain finite range, let us say $-z_c$, they can be neglected, and the broadening of a surface state will be governed by its overlap with this region of finite absorption

$$\delta E_{\perp} \propto \int_{-z_c}^{\infty} |\psi|^2 d^3r \quad (16)$$

the integral is always defined because $|\psi|^2$ decays away inside the crystal according to our assumptions for the existence of a surface state.

Consider a state corresponding to a large value of n . Being a high Rydberg state it will have many oscillations in the wavefunction near the barrier, and a maximum in amplitude at an approximate value of z given by

$$E_{\perp} = -\frac{1}{2} \frac{Z^2}{n^2} = -\frac{Z}{z_{\max}}, \quad (17)$$

$$z_{\max} = \frac{2n^2}{Z} \quad (18)$$

and the maximum amplitude A_{\max} will be given by the normalisation

$$\begin{aligned} |A_{\max}|^2 z_{\max} &\simeq 1 \\ A_{\max} &\simeq 1/\sqrt{z_{\max}} = \frac{1}{n} \sqrt{(Z/2)}. \end{aligned} \quad (19)$$

The mean amplitude in the immediate surface region is given by noting that whereas the mean kinetic energy of the state is

$$|Z^2/2n^2| \quad (20)$$

the kinetic energy near the surface is $\sim |Z/z_c|$, or greater, and therefore

$$\begin{aligned} |\psi|^2 &\leq |A_{\max}|^2 (\text{velocity at } z_c / \text{mean velocity})^{-1} \quad z > z_c \\ |\psi(z_c)|^2 &\simeq \frac{Z}{2n^2} \left(\frac{Z/z_c}{Z^2/2n^2} \right)^{-1/2} \\ &\propto n^{-3}. \end{aligned} \quad (21)$$

Therefore the broadening scales as n^{-3} and keeps pace with the level separation in the limit of large n . If we can see the first one or two members of a series, the rest should follow.

3. Analytic model for the Rydberg states

The model we adopt in the first instance is of a real band structure to represent the solid side and a z -dependent image potential, $V_B(z)$, with a cut-off, for the surface barrier:

$$V_B(z) = \begin{cases} (4z)^{-1}, & z \leq z_0 \\ -V_0 = (4z_0)^{-1}, & z \geq z_0. \end{cases} \quad (22)$$

We take the origin to be half an interlayer spacing outside the topmost atomic layer.

We shall specialise to the case when ψ_{\pm} are plane waves

$$\psi_{\pm} = |K_z|^{-1/2} \exp(i\mathbf{K}^{\pm} \cdot \mathbf{r}), \quad (23)$$

where

$$\mathbf{K}^{\pm} = (\pm K_z, \mathbf{k}_{\parallel}). \quad (24)$$

The condition for a surface state is

$$\phi_B + \phi_C = 2\pi n \quad (5)$$

and we begin by calculating ϕ_B .

In the absence of inelastic effects the image barrier reflectivity can be expressed in a standard manner in terms of the well known Whittaker functions, $W_{\lambda, \frac{1}{2}}(X)$ (Whittaker and Watson 1952) and is given by

$$\phi_B = -2\kappa z_0 - 2 \tan^{-1} [(2ik/\kappa) (W'_{\lambda, \frac{1}{2}}(\xi_0)/W_{\lambda, \frac{1}{2}}(\xi_0))] \quad (25)$$

where $\frac{1}{2}\kappa^2$ is the energy associated with motion normal to the surface measured relative to the zero inside the crystal,

$$\frac{1}{2}\kappa^2 = E - \frac{1}{2}|\mathbf{k}_{\parallel}|^2 + V_0 \quad (26)$$

and $\frac{1}{2}k^2$ is the same energy related to the energy zero in the vacuum

$$\frac{1}{2}k^2 = E - \frac{1}{2}|\mathbf{k}_{\parallel}|^2 = E_{\perp}, \quad (27)$$

$$\lambda = i/(4k), \quad (28)$$

$$\xi = 2ikz. \quad (29)$$

The prime in equation (25) denotes the derivative with respect to ξ and E_{\perp} is the energy associated with perpendicular motion. We use Wannier's (1943) approximation to the Whittaker functions in our calculations.

Now we allow for the possibility of creating surface excitations, this effect being represented by an imaginary component $\Sigma_i(z)$ in the surface potential. The one electron Schrödinger equation to be solved is then

$$-\frac{1}{2} \frac{d^2\psi}{dz^2} + (V_B(z) + i\Sigma_i(z))\psi = E\psi. \quad (30)$$

We assume that inelastic effects are small enough for the wavefunction of the localised state to remain unchanged from the non-absorption case. Absorptive effects, represented by $\Sigma_i(z)$ introduce an imaginary component into the phase of the barrier reflectivity, ϕ_{Bi} , which can easily be calculated by the same argument used to deduce equation (9). We get

$$\int_{-\infty}^{z_0} \Sigma_i(z) |\psi(z)|^2 dz = \frac{|A|^2 \kappa}{2} (\exp(-2\phi_{Bi}) - 1) \quad (31)$$

where $\psi(z) = W_{\lambda, \frac{1}{2}}(z)$ and A is a normalisation constant. The contribution to the integral coming from z_0 to 0 is negligible because $|\psi(z)|^2$ is very small in this region ($Z_0 \sim 0.2 \text{ \AA}$). Inkson (1972, 1973) has calculated the imaginary part of the surface self-energy using a Green function formulation of the many body problem. His result for a local-position, energy and momentum dependent self-energy is given by

$$\begin{aligned} \Sigma(z, p, w) &= \frac{i}{(2\pi)^4} \int G(z, \mathbf{p} - \mathbf{q}, w - w') W(z, \mathbf{q}, w') \\ &\times \exp(-i\delta w') dw' dq \end{aligned} \quad (32)$$

where G is the Green function, W the screened interaction and δ is an infinitesimal positive constant. Although Inkson's calculation is, we believe the best that has been done so far in this problem, the results are too complicated for practical purposes. The use of Inkson's imaginary part of the self-energy for the parallel case in equation (31) would be a considerable task clearly not justified by the rest of the approximations involved in the present work. A more simple expression for the imaginary part of the self-energy for this case has been derived by Echenique and Pendry (1975). Their result, given by

$$\Sigma_i(z) = -\frac{w_s}{2\sqrt{2E}} K_0\left(\frac{2w_s|z|}{\sqrt{2E}}\right) \quad (33)$$

(where K_0 is the Bessel function) is only exact in the limit of very high energies.

Equation (33) over-emphasises absorption and its use in equation (31) will produce unrealistic results for ϕ_{Bi} . We can correct for this deficiency of EP's semiclassical expression by shifting the z -origin of $\Sigma_i(z)$ inside the crystal. Although there are good physical reasons for justifying such a shift (Inkson 1972, 1973) our criterion will be to keep our $\Sigma_i(z)$ bigger than Inkson's one so that our calculation will provide an upper bound on

the energy broadening of the levels. (A typical shift is of around 1 Å for $E \sim 100$ eV).

In passing we note an interesting k_{\parallel} dependence of the broadening. As (k_{\parallel}) increases, longer wavelength excitations will be created and $\Sigma_i(z)$ will penetrate further into the vacuum (the range of penetration is $\simeq \sqrt{2E/w_s}$). A point will eventually be reached at which the effect of absorption is too big for the resonant state to have a meaning! Our calculations show that in our model (and for $w_s = 0.5$ au) this occurs at about $|k_{\parallel}| = 5$ au for the first resonant state. We estimate then that for energies about 10.0 H the first resonant state will be unresolved. If we keep increasing k_{\parallel} equation (31) will have a solution, for ϕ_{Bi} , for the first resonant state but not for the second (this occurs at $v = 13$ au in our model) and the same for all states.

We turn now to the crystal reflectivity. We shall assume that the phase of the reflectivity, ϕ_C , remains constant over the range of energies of the Rydberg states. This is certainly a good approximation for $n = 2$ onwards because ϕ_C varies through π across a band gap which is typically 0.1 H. There are two effects of which we must take account: the absorption in the crystal due to decay of the state into excitations of the crystal, and the fact that flux can also be lost by diffraction into other beams which escape from the surface, thus reducing r_C to being less than unity even in the absence of absorption due to inelastic effects. Both these processes we incorporate as an imaginary component to the phase

$$\phi_{Ci} \simeq -\ln r_C - V_{0i} \frac{\partial \phi_{Cr}}{\partial E} \quad (34)$$

where V_{0i} is the constant value taken by $\Sigma_i(z)$ inside the crystal and we have made a linear approximation to the complex-energy dependence of the phase.

The first term in the right-hand side of equation (34) relates to diffraction effects and the second to energy losses inside the crystal.

Finally, the condition for a surface state remains

$$\phi_C(E_{\perp}) + \phi_B(E_{\perp}) = 2\pi n \quad (5)$$

but as a consequence of the imaginary components of the phases defined in (31) and (34), now has solutions at complex energy, E_{\perp} , i.e. we have included the finite lifetime τ , of the surface state. Once again making a linear approximation to the complex energy dependence of ϕ_C and ϕ_B we deduce that

$$(1/2\tau)(\partial/\partial E)(\phi_{Br} + \phi_{Cr}) = \phi_{Ci} + \phi_{Bi} \quad (35)$$

where $(2\tau)^{-1}$ is the imaginary component of the energy that reproduces the required decay rate. It is the rapid variation of ϕ_{Br} near threshold that gives the long lifetimes scaling as n^{-3} to the higher Rydberg states. The variation of ϕ_C in the energy range we are interested in here (~ 0.05 H) is very small and in any case its inclusion would not much affect the resonant condition due to the rapid variation of the phase reflectivity with energy. We can take any reasonable value for it. We take an arbitrary value of $\phi_C = 1.6$ and neglect its variation with energy. The results of a model calculation for the first five resonant states are shown in table 1. As it can be seen, the resonant levels, i.e. the solutions of equation (5) fit into a Rydberg series of the form

$$E_{\perp n} = -[32(n+a)^2]^{-1} \quad (36)$$

with $a = 0.21$. We include the 'hydrogenic levels' ($a = 0$) for comparison.

Table 1. Resonant states energies (Hartrees) and their energy broadening. Model parameters: $\phi_C = 1.6$; $E_{gap} = 0.15$ au; $V_0 = 0.6$ au; $w_s = 0.5$ au; $V = 2$ au.

$-E_B$ (Hydrogenic)	$-E_{\perp}$ (Resonant states)	$d\phi_B/dE$	$\delta E_{\perp} = 1/(2\tau)$
3.12×10^{-2}	2.13×10^{-2}	2.78×10^2	1.19×10^{-2}
7.81×10^{-3}	6.39×10^{-3}	1.67×10^3	1.93×10^{-3}
3.47×10^{-3}	3.03×10^{-3}	5.10×10^3	5.53×10^{-4}
1.95×10^{-3}	1.76×10^{-3}	1.12×10^4	2.57×10^{-4}
1.25×10^{-3}	1.15×10^{-3}	2.18×10^4	1.26×10^{-4}

4. Experimental observation

In order to be able to measure effects associated with surface resonances a trapped electron has to escape from such a state so it can be detected after escaping into the vacuum. Diffraction experiments provide a useful way of studying such states. The resonant states are coupled to the incident beam, and to other beams, which can escape to the vacuum, in which they create fine structure. We shall work in the 'four-beam' approximation (McRae 1971), which is the simplest model including the essential physics of the problem: two beams for the surface state, two other beams with different k_{\parallel} for the incident and reflected beams. In the 'four-beam' approximation the reflection amplitude is given (in our notation) by McRae (1971)

$$R = S_{00} + S_{0g} \frac{\exp(i\phi_B)}{1 - \exp i(\phi_C + \phi_B)} S_{g0} \quad (37)$$

where S_{00} is the direct reflection amplitude and S_{g0} and S_{0g} represent coupling into and out of the surface state beams.

The resonances correspond to zeros of $[1 - \exp i(\phi_B + \phi_C)]$. The inclusion of lifetime effects discussed in the previous section will broaden the resonances. In the vicinity of a given resonance, $E_{\perp n}$, the amplitude will be given, in a pole approximation, by

$$R = S_{00} + C_n \frac{1/(2\tau_n)}{E - E_{\perp n} + i/(2\tau_n)} \quad (38)$$

where C_n represents the overall amplitude of the resonant contribution with respect to the background and is given by the residue of the poles in equation (37)

$$C_n \simeq S_{0g} S_{g0} \frac{2\tau_n \exp(i\phi_B)}{[-i(\partial/\partial E)(\phi_C + \phi_B)]} \quad (39)$$

Substituting from (35) we deduce

$$\begin{aligned} C_n &= i S_{0g} S_{g0} \frac{2\tau_n \exp(i\phi_B)}{2\tau_n(\phi_{Ci} + \phi_{Bi})} \\ &= i S_{0g} S_{g0} \frac{\exp(i\phi_B)}{\phi_{Ci} + \phi_{Bi}} \end{aligned} \quad (40)$$

Thus, except for the phase factor, C_n tends to a constant as n increases, and we expect to see structure in the LEED reflectivity oscillating increasingly rapidly towards threshold,

with constant amplitude. Figure 3 shows an evaluation of (37) confirming this conclusion. Note that although the energy levels are broadened they still remain visible.

One further consequence of formula (37) should be noted. Non-surface state resonant LEED spectra are subject to a uniform broadening of between 0.1 and 0.4 H by the strong inelastic scattering in the crystal. All diffraction that occurs within the crystal is broadened to this extent, but the resonant scattering escapes these processes to the extent that the Rydberg states involved lie far out *in vacuo*. Thus a Rydberg series will show structure

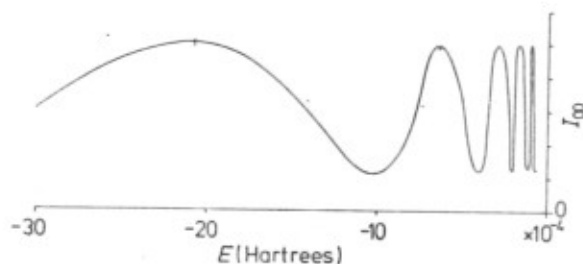


Figure 3. LEED intensity profiles (in arbitrary units) for the first five resonant states with the background subtracted. In abscissa we plot the energy difference from the emergence condition.

as fine as 0.001 H (30 meV) in the fourth member of the series. This very fine structure serves to make a definitive identification of Rydberg resonances in LEED, but of course also poses difficulties for the experimentalist in obtaining the high resolution necessary.

Finally we should like to add that if it proves possible to detect several members of the Rydberg series, and measure their energies and widths, a most valuable probe of the surface barrier will have been established. Little has been deduced by direct experiment as yet about the detailed shape of the barrier which will dictate the details of the energy levels. In particular the variation of inelastic scattering with distance from the surface would be probed by the variation of peak width with n .

5. Conclusions

We have shown that the Rydberg spectrum of surface resonances at solid surfaces can be resolved, the rapid variation of the phase of the image potential reflectivity being the main reason for this result. We have shown that inelastic surface scattering effects are not strong enough to prevent the resonant states being resolved over most of the LEED energy range. We predict a given velocity parallel to the surface, i.e. a given energy for which the states will begin to be unresolved, 'reappearing' again gradually as the velocity increases. A detailed experimental study over a wide range of energies of these problems (via LEED) will bring much experimental information about the real and imaginary part of the surface potential which has been lacking to date. We hope that our work will provoke such experiments.

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